

# Realistic SUSY Model with Four Fermion Families, Natural R Parity and $\nu_\tau$ in the eV Range

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## Abstract

We study an extension of the supersymmetric standard model with four families and gauged horizontal symmetry  $SU(4)_H$ , in which R parity automatically follows as a consequence of gauge invariance and of the field content of the model. Beyond ensuring R parity conservation, the horizontal symmetry allows for the dynamical generation of a hierarchical pattern of fermion masses, without the need of any ad hoc choice of small Yukawa couplings. The structure of the mass matrices implies that the fourth family does not mix with the other three, and that the  $b', t', \tau', \nu'$  masses are all naturally in the 100 GeV range. The model has other interesting phenomenological implications. The scale of the horizontal symmetry breaking is constrained by cosmological and astrophysical arguments to be  $\sim 10^{11}$  GeV. Then the late  $b'$  decays could explain a magnitude of the isotropic cosmic gamma flux. In addition, a lower bound of a few eV is found for the mass of the  $\tau$ -neutrino, which can thus provide a hot component for the cosmological dark matter. Due to R parity conservation, the lightest supersymmetric particle is stable and can provide the cold component. The  $\nu_e$  and  $\nu_\mu$  masses can naturally be in the correct ranges for a solution of the solar or atmospheric neutrino problems via neutrino oscillations.

# 1. Introduction

In the Standard Model (SM), Baryon (B) and Lepton (L) numbers are conserved as a result of accidental global  $U(1)_B$  and  $U(1)_L$  symmetries that follow from the requirement of gauge invariance and renormalizability.<sup>1</sup> In the supersymmetric standard model (SSM) this is not true anymore. In the superpotential, the renormalizable terms

$$lle^c, \quad lqd^c, \quad u^c d^c d^c. \quad (1)$$

( $l$ ,  $e^c$  and  $q$ ,  $u^c$ ,  $d^c$  are the lepton and quark superfields) are allowed by the gauge symmetry, which violate L and B. In particular, the combination of first and third of these terms would lead to catastrophically fast proton decay mediated by  $d^c$ -type squark exchange. The relevant symmetry that ensures the absence of the terms (1) is called R parity, which is defined as  $R \equiv (-1)^{2J+3B+L}$ , where J is the spin of the particle and B(L) its baryon (lepton) number [2]. R parity does not commute with supersymmetry (SUSY). On the other hand, it is an automatic consequence of a  $Z_2$  matter parity under which the fermion superfields change the sign while the ‘Higgs’ ones  $\Phi_{1,2}$  remain invariant.

An unsatisfactory feature of the SSM is that the  $Z_2$  (or equivalently R) parity conservation has to be imposed by hand. The SUSY  $SU(5)$  model does not differ much from the SSM, since the gauge invariant term  $\mathbf{10} \bar{\mathbf{5}} \bar{\mathbf{5}}$ , involving the fermion superfields in the  $\mathbf{10} + \bar{\mathbf{5}}$  representation of  $SU(5)$  leads again to the set of B and L violating couplings (1). the  $SO(10)$  GUT offers an elegant solution to this problem: the Higgs fields are generally assigned to vector representations ( $\mathbf{10}$ ,  $\mathbf{45}$ ,  $\mathbf{54}$ ,  $\mathbf{126}$  etc.) while the fermion superfields are in the spinor representation  $\mathbf{16}$ . While all the needed mass terms are allowed, the terms in (1) would arise from  $\mathbf{16}^3$  which is forbidden by the gauge symmetry. In other words, since  $SO(10)$  invariance allows for only pairs of 16-plets, the theory has an automatic  $Z_2$  matter parity under which 16-plets change the sign whereas the superfields in vector representations remain invariant. However, this is not true anymore for the  $SO(10)$  models in which symmetry breaking is triggered also by the scalar components of superfields belonging to the  $\mathbf{16}_\Phi + \bar{\mathbf{16}}_\Phi$  [3]. In fact, after substituting the VEV  $\langle \mathbf{16}_\Phi \rangle$ , the couplings  $\frac{1}{M} \mathbf{16}^3 \mathbf{16}_\Phi$  lead again to the terms (1). Since in these models the right handed neutrino masses are generated by operators  $\sim \frac{1}{M} \mathbf{16}^2 \mathbf{16}_\Phi^2$  of the same structure, the ratio  $\langle \mathbf{16}_\Phi \rangle / M$  cannot be too small, implying in turn that the magnitude of the resulting R-parity violating terms conflicts again with the limits on the proton lifetime. Thus R-parity conservation is not automatic in  $SO(10)$  models with Higgses belonging to the  $\mathbf{16}_\Phi$ , and matter parity has to be imposed by hand in order to distinguish the fermion 16-plets from the Higgs ones.

In this paper we wish to put forward the idea that R (or equivalently  $Z_2$ ) parity conservation can be naturally ensured in models based on *gauged* non-Abelian horizontal symmetries. Such models are particularly interesting since they can also explain at least qualitatively the observed pattern of fermion masses and mixing. Namely, the structure of the fermion mass matrices can be related to the horizontal symmetry breaking pattern, and the mass hierarchy between families arises dynamically from certain hierarchies in this

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<sup>1</sup>These symmetries can be broken only by non-renormalizable Planck scale operators like  $\frac{1}{M} ll\Phi\Phi$  [1]. This term violates L and induces the neutrino Majorana masses of about  $10^{-5}$  eV, which could be relevant for the solar neutrino oscillations. However, analogous terms violating baryon number are very small to cause any observable effect.

breaking (see for example the models [4, 5] based on  $SU(3)_H$  symmetry). In this kind of models it is natural to assume that the horizontal group  $G_H$  acts only on the quark-lepton superfields, while the Higgses  $\Phi_{1,2}$  responsible for the electroweak symmetry breaking are  $G_H$ -singlets. Hence, independently of the choice of the vertical gauge group and/or of the particular superfield assignments to its representations, the Higgs and fermion superfields can be always distinguished since they carry different horizontal quantum numbers. This leads to the possibility of allowing the necessary mass terms which are bilinear in the fermion superfields, while forbidding the trilinear couplings (1).

Our task is now to find and classify the theories in which the horizontal gauge group  $G_H$  automatically forbids the unwanted B and L violating terms in (1). Then R parity (or equivalently  $Z_2$  matter parity) will appear as an automatic consequence of the horizontal gauge symmetry and of the field content of the model. We demand that the models we are interested in should satisfy the following list of basic requirements:

(i) In order to ensure a straightforward definition of the horizontal gauge symmetry, all the fermion superfields  $q, l, u^c, d^c$  and  $e^c$  should be assigned to the same representation of the non-Abelian horizontal group  $G_H$ . In other words, we forbid  $G_H$  singlet families.

(ii) We require that the couplings in (1) are forbidden by gauge invariance. Therefore  $SO(3)_H$  and chiral  $SU(3)_H$ , which are the only simple groups containing three dimensional representations, are excluded since in both cases  $\mathbf{3}^3$  contains a gauge singlet. However we immediately notice that for  $SU(N)_H$  ( $N > 3$ ) the term  $N^3$  does not contain gauge singlets. Hence these groups represent a class of interesting candidates.

To have phenomenologically realistic theories, the following additional constraints should be also imposed:

(iii) In order to avoid the proliferation of Higgs doublets with masses at the electroweak scale, the standard Higgs superfields  $\Phi_{1,2}$  must be  $G_H$  singlets. The presence of several Higgs doublets would in fact spoil the natural suppression of Flavor Changing Neutral Currents (FCNC). It would also destroy gauge coupling unification, thus preventing any attempt to embed the model in some vertical GUTs.

(iv) The model should provide naturally a realistic pattern of the fermion mass and mixing. In this respect, models based on chiral  $SU(3)_H$  [4, 5] have proven to be quite successful in relating the fermion mass hierarchy to a hierarchy in the horizontal symmetry breaking VEVs, and in accounting for the observed fermion mass and mixing pattern. Even if chiral  $SU(3)_H$  fails to satisfy condition (ii), this again suggests that chiral  $SU(N)_H$  ( $N > 3$ ) are promising candidates.<sup>2</sup>

(v) A final strong condition is that R-parity breaking terms should not appear even after  $G_H$  breaking. In other words, no effective couplings which could generate the terms in (1) after substituting the VEVs  $\xi^k \rightarrow \langle \xi^k \rangle$  should be allowed by the  $G_H$  symmetry. In particular, this condition restrict the viable  $SU(N)_H$  models to even  $N$ . Consider

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<sup>2</sup> We note that vectorlike  $SU(3)_H$  with  $q, l$  transforming as  $\mathbf{3}$  and  $u^c, d^c, e^c$  as  $\bar{\mathbf{3}}$ , forbids the first two terms in (1), which is enough to ensure proton stability. However it allows for the  $SU(3)_H$  invariant Yukawa terms  $f f^c \Phi_{1,2}$ . Then the mass splitting between different families can be achieved only at the price of *several* unnatural fine tunings between horizontal non-invariant effective operators. Thus this case can be hardly regarded as realistic. (In addition unification of the quarks and leptons within one irreducible GUT multiplet would clearly be not possible). The same problem is encountered for  $SO(4)$  and  $SO(5)$  with 4 dimensional representations. While the  $\mathbf{4}^3$  terms are forbidden and condition (ii) is fulfilled, the invariant Yukawa terms  $f f^c \Phi_{1,2}$  are again allowed, and thus condition (iv) is not satisfied.

in fact  $SU(N)_H$  with the  $f$  and  $f^c$  fermion superfields assigned to the fundamental  $N$  dimensional representation. The mass terms transform as  $N \times N$  and thus belong to two-index (symmetric and antisymmetric) representations. In order to construct horizontal gauge invariant mass terms we can take also the horizontal Higgses  $\xi^k$  in two-index representations. Then for  $N=4,6,\dots$  no invariants of the form  $N \times N \times N \times \mathcal{P}(\xi)$  (with  $\mathcal{P}$  some polynomials in the horizontal fields  $\xi^k$ ) are allowed. In contrast, for  $N$  odd the totally antisymmetric  $\epsilon$  tensor allows to rewrite some combinations of Higgs fields with an even number of free indices as tensors with an odd number of free indices, suitable for generating effective  $N \times N \times N \times \mathcal{P}(\xi)$  gauge invariant terms. After the breaking of the horizontal symmetry ( $\xi^k \rightarrow \langle \xi^k \rangle$ ) these effective operators will then produce the R-violating terms in (1).

This brief analysis suggests that natural conservation of R-parity could be achieved in models based on chiral horizontal symmetries  $SU(N)_H$  with  $N$  even, under which the quark and lepton superfields transform as fundamental  $N$ -plets. Hence the number of families must be extended to  $N_f = 4, 6, \dots$ . As is well known, the possibility of extra families with a light neutrino is ruled out by the measured invisible decay width of  $Z$ -boson, however, sequential generations with heavy neutrinos ( $m > M_Z/2$ ) are not excluded. On the other hand, detailed studies [6] of the effects of radiative corrections due to additional families show that precise electroweak data are compatible with a fourth family, while six families (which would be the next interesting case) are ruled out. In addition, a dedicated analysis showing the viability of SUSY models with four families with respect to gauge coupling unification was presented in ref. [7]. These results are relevant for our analysis, since condition (iii) ensures that the field content in our  $SU(4)_H$  model is the same than that of the four family SSM of ref. [7], up to some large energy scale where the horizontal symmetry breaks down (see Sect. 2). Hence, we conclude that if natural R parity conservation arises due to some non-Abelian horizontal gauge symmetry, then the theoretical and phenomenological constraints hint to models based on the  $SU(4)_H$  group, on which we will concentrate in the rest of the paper.

## 2. $SU(4)_H$ symmetry and its phenomenological implications

Let us consider the standard  $SU(3) \times SU(2) \times U(1)$  vertical gauge group, with local chiral  $SU(4)_H$  horizontal symmetry acting on four families of left chiral superfields

$$\begin{aligned} f_\alpha : \quad q_\alpha = \begin{pmatrix} u \\ d \end{pmatrix}_\alpha &\sim (3, 2, 1/6)_\alpha, & l_\alpha = \begin{pmatrix} \nu \\ e \end{pmatrix}_\alpha &\sim (1, 2, -1/2)_\alpha \\ f_\alpha^c : \quad u_\alpha^c &\sim (\bar{3}, 1, -2/3)_\alpha, & d_\alpha^c &\sim (\bar{3}, 1, 1/3)_\alpha, & e_\alpha^c &\sim (1, 1, 1)_\alpha \end{aligned} \quad (2)$$

where each superfield is assigned to the fundamental  $\mathbf{4}$  representation ( $\alpha = 1, \dots, 4$  is the  $SU(4)_H$  index). With this field content the horizontal  $SU(4)_H$  is anomalous. In order to cancel the horizontal anomaly we introduce the following superfields which are vectorlike with respect to  $SU(3) \times SU(2) \times U(1)$  and belong to the  $\bar{\mathbf{4}}$  of  $SU(4)_H$ :

$$\begin{aligned} F^\alpha : \quad U^\alpha &\sim (3, 1, 2/3)^\alpha, & D^\alpha &\sim (3, 1, -1/3)^\alpha, & E^\alpha &\sim (1, 1, -1)^\alpha \\ F_c^\alpha : \quad U_c^\alpha &\sim (\bar{3}, 1, -2/3)^\alpha, & D_c^\alpha &\sim (\bar{3}, 1, 1/3)^\alpha, & E_c^\alpha &\sim (1, 1, 1)^\alpha, & N_c^\alpha &\sim (1, 1, 0)^\alpha \end{aligned} \quad (3)$$

As we will see in short, these superfields turn out to be necessary also for providing masses to the known fermions.

In the Higgs sector, we choose the standard Higgs doublet superfields  $\Phi_{1,2}(1, 2, \mp 1/2)$  to be singlets under  $SU(4)_H$ . In order to break the horizontal symmetry and to generate the fermion masses, we introduce also a set of ‘horizontal’ Higgs superfields  $\xi_{\{\alpha\beta\}}^k$  ( $k = 1, 2, \dots$ ) transforming as the symmetric **10** representations of  $SU(4)_H$  (clearly, they must be  $SU(3) \times SU(2) \times U(1)$  singlets). Additional superfields  $\bar{\xi}^k$  transforming as  $\overline{\mathbf{10}}$  are also needed to render the Higgsino sector free of chiral anomalies. With this field content, the most general Yukawa terms allowed by gauge invariance read as

$$W_F = g_f f_\alpha F_c^\alpha \Phi_{1(2)} + \sum_k h_F^k F^\alpha F_c^\beta \xi_{\alpha\beta}^k + \Lambda_f F^\alpha f_\alpha^c, \quad F = E, D, (U) \quad (4)$$

$$W_N = g_\nu l_\alpha N_c^\alpha \Phi_2 + \sum_k h_N^k N_c^\alpha N_c^\beta \xi_{\alpha\beta}^k \quad (5)$$

where the  $g$ ’s and  $h$ ’s are Yukawa couplings which we take to be all  $O(1)$ . The last term in eq. (4) is a gauge invariant bilinear, and the  $\Lambda_f$ ’s are some large mass parameters. As already stated, no terms trilinear in the quark and lepton superfields are allowed by the  $SU(4)_H$  gauge symmetry, ensuring naturally the absence of the B and L violating couplings (1). We are facing here a situation analogous to the  $SO(10)$  model, since R-parity does not have to be imposed by hand, but appears as an accidental symmetry that follows from horizontal gauge invariance. The terms (4), (5) are in fact invariant with respect to the  $Z_2$  parity under which the fermion superfields  $f, f^c, F, F_c$  change sign, while the Higgs superfields  $\Phi_{1,2}$  and  $\xi$  stay invariant. More in general, these have an automatic global symmetry  $U(1)_H$  under the following transformations:

$$f, f^c \rightarrow e^{i\omega} f, f^c, \quad F, F^c \rightarrow e^{-i\omega} F, F^c, \quad \xi^k \rightarrow e^{2i\omega} \xi^k, \quad \Phi_{1,2} \rightarrow \Phi_{1,2} \quad (\bar{\xi}^k \rightarrow e^{-2i\omega} \bar{\xi}^k) \quad (6)$$

Its  $Z_2$  subgroup ( $\omega = \pi$ ) remains unbroken even when the scalars  $\xi$  get non-zero VEVs. Clearly, this  $Z_2$  matter parity which ensures R parity conservation and hence proton stability will be respected by all possible higher order Planck scale operators as well.

The Yukawa terms (4) lead to the so called “universal seesaw” mechanism [8, 9] for generating the masses of the charged fermions, while the terms (5) lead to the ordinary seesaw mechanism for neutrino masses [10]. After the horizontal scalars  $\xi^k$  develop non-zero VEVs, the extra fermions  $F$  and  $F_c$  of eq. (3) acquire large masses through the second term in eq. (4). Then the first and third terms cause a “seesaw” mixing of the ordinary quarks and leptons  $f, f^c$  with the heavy ones. As a result, in the base  $(f, F)$  ( $f^c, F_c$ ), the  $8 \times 8$  mass matrix for the charged fermions  $f = e, d, (u)$ ,  $F = E, D, (U)$  reads

$$\mathcal{M}_f = \begin{pmatrix} 0 & g_f v_{1(2)} \\ \Lambda_f & \hat{M}^F \end{pmatrix}, \quad \hat{M}_{\alpha\beta}^F = \sum_k h_F^k \langle \xi_{\alpha\beta}^k \rangle \quad (7)$$

where  $v_{1,2} = \langle \tilde{\Phi}_{1,2} \rangle$  are the VEVs of the two electroweak Higgs doublets. As for the neutral states, in the base  $(\nu, N_c)$  the  $8 \times 8$  Majorana mass matrix has the form

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & g_\nu v_2 \\ g_\nu v_2 & \hat{M}^N \end{pmatrix}, \quad \hat{M}_{\alpha\beta}^N = \sum_k h_N^k \langle \xi_{\alpha\beta}^k \rangle. \quad (8)$$

In contrast to the SM and most GUT models, in our picture the fermion mass hierarchy is not generated by an *ad hoc* choice of the Yukawa couplings. In fact we can assume all the Yukawa constants to be  $O(1)$ , close to the size of the gauge couplings. As long as the off-diagonal blocks in eqs. (7) and (8) are flavour blind (unit) matrices, all the informations on the fermion mass and mixing pattern is contained in the heavy mass matrices  $\hat{M}^F$ . The structure of the latter is determined by the different VEVs  $\langle \xi^k \rangle$  (modulo differences in the Yukawa constants  $h_F^k$ ), and the observed hierarchy of the light fermion masses is ultimately determined by the hierarchy in the VEVs which break the  $SU(4)_H$  symmetry. In other words, the VEV pattern should provide a step-by-step breaking of the chiral horizontal symmetry

$$SU(4)_H \times U(1)_H \xrightarrow{V_1} SU(3)_H \times U(1)'_H \xrightarrow{V_2} SU(2)_H \times U(1)''_H \xrightarrow{V_3} \tilde{U}(1). \quad (9)$$

The first breaking at the scale  $V_1$  (given by the VEVs  $\langle \xi_{11}^k \rangle$ ) provides the mass terms for the first heavy family  $F_1$ , the second breaking (at  $V_2 \sim \langle \xi_{\alpha 2}^k \rangle$ ,  $\alpha \leq 2$ ) for the second family  $F_2$  and the third stage of breaking at  $V_3 \sim \langle \xi_{\alpha 3}^k \rangle$ ,  $\alpha \leq 3$  generates the mass of the  $F_3$  state. At this stage all the horizontal gauge bosons have acquired large masses, the less massive are the ones responsible for the flavour-changing transitions between the fourth and the third families, with masses  $\sim V_3$ . The model also provides a natural possibility to obtain together with three light families, a heavy fourth, for which all the masses are of the order of the electroweak scale. In fact let us assume that all the VEVs of the type  $\langle \xi_{\alpha 4}^k \rangle$  are vanishing, so that a diagonal global  $\tilde{U}(1)$  subgroup of  $SU(4)_H \times U(1)_H$ , given by the generator  $\tilde{T} = \text{diag}(0, 0, 0, 1)$ , is left unbroken.<sup>3</sup> Then the  $4 \times 4$  mass matrices  $\hat{M}^{F(N)}$  are *rank-3* matrices of the following form

$$\hat{M}^F = \begin{pmatrix} M_F^{(3)} & 0 \\ 0 & 0 \end{pmatrix}, \quad F = U, D, E, N \quad (10)$$

where the  $3 \times 3$  blocks  $M_F^{(3)}$  contain non-zero entries, given by the VEVs of the horizontal scalars. Clearly, the residual global symmetry  $\tilde{U}(1)$  ensures naturally the heaviness of the fourth family, since there is no seesaw mechanism for the corresponding fermions. The right-handed components of the fields  $f_4 = b', t', \tau', \nu'$  are actually the  $F_c^4$  states, whereas the  $f_4^c$  form with the  $F_4$  superheavy particles of mass  $\Lambda_f$ . From eqs. (7) and (8) we obtain

$$m_{b', \tau'} = g_{d,e} v_1, \quad m_{t', \nu'} = g_{u,\nu} v_2. \quad (11)$$

Since the Yukawa couplings are all  $O(1)$ , for moderate values of  $\tan \beta = v_2/v_1$  all the masses in (11) are of the order of electroweak scale  $\sim 100$  GeV. On the other hand, it is apparent from (7) that the fermions of the first three families will acquire their masses through a seesaw mixing with the superheavy  $F$  fermions. Their masses will then be suppressed from the electroweak scale down to the observed values, provided that  $M_F^{(3)} > \Lambda_f$ . Namely, after decoupling the heavy states<sup>4</sup> the  $3 \times 3$  mass matrices of the light

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<sup>3</sup>A detailed analysis of the horizontal Higgs potential will be given elsewhere.

<sup>4</sup> Below the scale  $V_H = V_3$  our theory simply reduces to the SSM with four families. Therefore, for  $V_H$  large enough all FCNC phenomena related to the horizontal gauge or Higgs bosons and to the mixing with weak isosinglet heavy fermions are strongly suppressed.

charged fermions  $f = e, d, (u)$  are  $m_f^{(3)} = g_f \mu_f (M_F^{(3)})^{-1} v_{1(2)}$  while for the  $3 \times 3$  Majorana mass matrix for the light neutrinos we obtain  $m_\nu^{(3)} = (M_N^{(3)})^{-1} (g_\nu v_2)^2$ . Hence through the seesaw mechanism the horizontal VEV pattern (9) is reflected in the observed pattern of fermion masses in an inverted way [8] (see also [13]). Modulo the different  $O(1)$  Yukawa constants  $h_F^k$  the heavy mass eigenstates of  $M_F^{(3)}$  reflect the hierarchy  $V_1 \gg V_2 \gg V_3$  of the  $SU(4)_H$  symmetry breaking. On the other hand, the ordinary quark and lepton masses are inversely proportional to the masses of their heavy partners. For example, for the charged lepton and the neutrino masses we have<sup>5</sup>

$$m_{e,\mu,\tau} \simeq \frac{\Lambda_e m_{\tau'}}{h_E^{1,2,3} V_{1,2,3}} < m_{\tau'} , \quad m_{\nu_e, \nu_\mu, \nu_\tau} \simeq \frac{m_{\nu'}^2}{h_N^{1,2,3} V_{1,2,3}} \ll m_{\nu'} \quad (12)$$

The quark masses are given by expressions analogous to that of charged leptons. The CKM matrix of mixing between the first three families is also determined by the structure of the matrices  $M_F^{(3)}$ , and arises from the presence of non-diagonal VEVs  $\langle \xi_{\alpha\beta}^k \rangle$  ( $\alpha, \beta = 1, 2, 3$ ). The fact that the  $b$  and  $\tau$  masses are of order a few GeV, implies that the masses of the corresponding heavy states  $h_D^3 V_3$  and  $h_E^3 V_3$  are not much larger (say, within one or two orders of magnitude) than the mass scale  $\Lambda_{d,e}$ . As for the top quark, the value of its mass  $m_t \gtrsim 150$  GeV requires  $h_U^3 V_3 \sim \Lambda_u$ . In this case corrections to the seesaw formula should be taken into account in relating  $m_t$  to the heavy scales (see e.g. [13]).

As we stated above, the residual  $\tilde{U}(1)$  symmetry implies that fourth neutrino  $\nu'$  is a Dirac particle with mass  $\sim 100$  GeV, while three neutrinos get small Majorana masses given by eq. (12). Therefore, the neutrino mass hierarchy is expected to be approximately the same as the hierarchy between the quarks or the charged leptons:

$$m_{\nu_e} : m_{\nu_\mu} : m_{\nu_\tau} \sim m_u : m_c : m_t \quad \text{or} \quad \sim m_e : m_\mu : m_\tau. \quad (13)$$

On the experimental side, the firmest constraints on the masses of any new sequential fermion, quark or lepton, have been set at LEP:  $m_f \gtrsim M_Z/2$ . This indeed represents the best constraint on  $m_{\tau'}$  and  $m_{\nu'}$ . Searches for new quarks at the TEVATRON collider could in principle give much better bounds for  $m_{t'}$  and  $m_{b'}$  [11]. However, let us note that the structure of the heavy mass matrix (10) implies that the fourth family is unmixed with the three lighter ones. Hence the usual signatures, as for example  $b' \rightarrow c, u$ , that have been used to set the limits on new sequential quarks [11] do not occur in our case. In the absence of a detailed experimental analysis of the unmixed case, the only reliable limit is again the LEP one also for the new quarks. Hence we can safely conclude that the predictions in (11) are by no means in conflict with the existing experimental limits. However, it is clear that for the masses of the fourth family fermions not much room is left. The allowed parameter space is strongly constrained by the CDF measurement of the top mass,  $m_t = 174 \pm 10 \pm 13$  GeV [12], by the precision tests of the SM which do not leave much space for additional sizeable radiative corrections as would be induced by a too large  $m_{t'} - m_{b'}$  splitting, and by renormalization group analysis of the Yukawa couplings, much in the spirit of ref. [7]. In particular, the universal seesaw mechanism implies  $m_{t'} \geq m_t$ , and most likely  $m_{t'} \geq m_{b'}$ . Then, according to [7], for  $m_{t'} \geq m_t > 150$  GeV the

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<sup>5</sup>Since in the following we deal with order of magnitude estimates, we neglect the different  $O(1)$  factors accounting for the renormalization group running of masses from the horizontal scale to lower energies.

consistency of the model implies not too large masses for the fourth family fermions. For the low values of  $\tan\beta$  we are interested in (e.g.  $\tan\beta \sim 2$ ), the maximal values allowed are about  $m_{b'} \sim 100$  GeV and  $m_{\tau',\nu'} \sim 50$  GeV, that is within the reach of LEP II.

### 3. Astrophysical consequences of the model

Indeed the presence of a fourth family of fermions constitutes the most compelling prediction of our model, so we will now address some issues regarding these new states. The unbroken diagonal  $\tilde{U}(1)$  subgroup of  $SU(4)_H \times U(1)_H$  implies that the fourth family is unmixed with the three lighter ones. We assume that the lightest member of the fourth generation is the neutral one  $\nu'$ , as is also suggested by the analysis of ref. [7]. For simplicity we also assume that  $m_{\nu'} > m_{b'}$ . Then  $b'$  and  $\nu'$  are stable with respect to electroweak interactions. A stable  $\nu'$  with mass in the 100 GeV range is cosmologically safe, the contribution  $\rho_{\nu'}$  to the present cosmological density would not exceed the critical density  $\rho_c$  even in the presence of a  $\nu'$ - $\bar{\nu}'$  primordial asymmetry of the order of the baryon asymmetry. However, as we will argue in the following, in our model no sizeable asymmetry has to be expected for the fourth family fermions.

In contrast, the existence of stable heavy quarks carrying colour and electric charge would constitute serious problem, since it will conflict with the constraints arising from superheavy element searches [15]. Indeed, the stable  $b'$  would behave essentially as  $d$  quarks, hadronising into heavy stable mesons  $\bar{b}'u$  and ‘nucleons’  $b'ud$  [15] giving rise to heavy hydrogen-like ‘isotopes’ with masses  $\sim 100$  GeV. The existing experimental limits on this kind of isotopes are extremely tight. For  $m_{b'} < 1$  TeV the limit on the  $b'$  abundance relative to normal hydrogen is  $n_{b'}/n_B < 10^{-28}$  [16].

However, the exchange of the  $SU(4)_H$  gauge bosons  $Z_H$  would allow the heavy quark to decay, dominantly through the channel  $b' \rightarrow b\bar{\nu}_\tau\nu'$ , with a lifetime

$$\tau_{b'} \simeq \left(\frac{V_H}{v}\right)^4 \left(\frac{m_\mu}{m_{b'}}\right)^5 \tau_\mu = \left(\frac{V_H}{10^{12} \text{ GeV}}\right)^4 \left(\frac{150 \text{ GeV}}{m_{b'}}\right)^5 \cdot 4 \cdot 10^{17} \text{ s} \quad (14)$$

where  $V_H = V_3$  is the lowest scale in the horizontal symmetry breaking (see eq. (9)),  $v = 174$  GeV is the electroweak scale and  $\tau_\mu = \tau(\mu \rightarrow e\bar{\nu}_e\nu_\mu)$  is the muon lifetime. We can use cosmological arguments, together with the experimental limits on searches for heavy isotopes, to put an upper bound on  $\tau_{b'}$ , which in turn will translate in an upper limit on  $V_H$ . Taking into account the finite lifetime of the heavy quarks, their present number abundance relative to baryons is  $n_{b'}/n_B = r_0 \exp(-t_0/\tau_{b'}) \leq 10^{-28}$ , where  $r_0 = (n_{b'}/n_B)_0$  is the relic abundance for stable  $b'$ ,  $h = 0.5 - 1$  is the Hubble parameter in units of  $100 \text{ Km s}^{-1} \text{ Mpc}^{-1}$ , and  $t_0 \simeq 2 \cdot 10^{17} h^{-1} \text{ s}$  is the present age of the Universe (we assume matter dominated expansion of the Universe, and  $\Omega = 1$  as it is motivated by inflation). From this equation we get an upper limit on the  $b'$  lifetime

$$\tau_{b'} \leq 3.1 \cdot 10^{15} h^{-1} (1 + 0.036 \lg r_0)^{-1} \text{ s}. \quad (15)$$

Due to many theoretical uncertainties related to the actual annihilation cross section for the  $b'$ , it is not possible to compute precisely the value of  $r_0$ . However, an estimate of the relic abundance of heavy stable  $d$ -type quarks has been given in [15]. Under the

assumption that there is no cosmological  $b'$  and  $\bar{b}'$  asymmetry, it was found that for  $m_{b'} \sim 150 \text{ GeV}$  the energy density of these relics, relative to critical density (namely  $\Omega_{b'} h^2$ ) could range from  $10^{-9}$  to  $10^{-4}$  (smaller values are obtained for lighter  $b'$  masses). The lower limit corresponds to the case when the relic density is determined by the annihilation after the QCD phase transition, and it was obtained by taking as an upper bound on the annihilation cross section the geometrical cross section ( $\sigma_0 \sim 100 \text{ mb}$ ). The upper limit was obtained under the opposite assumption, namely that annihilation after confinement is negligible, and that the relic density is essentially determined by the QCD annihilation cross section for free quarks. Then the ratio of the  $b'$  to baryon number densities  $r_0 = (n_{b'}/n_B)_0 = \Omega_{b'}/\Omega_B \cdot m_B/m_{b'}$  lies in the range  $3 \cdot 10^{-10} < r_0 < 3 \cdot 10^{-5}$  where we have taken  $\Omega_B \sim 0.02$  as suggested by nucleosynthesis estimates. As is discussed in [15], the most reasonable assumption is that the relevant annihilation process happens after confinement, however with a cross section much smaller than the geometrical one, giving  $r_0 \sim 10^{-7} - 10^{-8}$ . Clearly, in the presence of a sizeable baryon asymmetry in the fourth family sector, the relic abundance of the heavy  $b'$  quarks would be some orders of magnitude larger than the quoted estimates, up to  $r_0 \sim 1$ .

As we see, the bound (15) very weakly depends on the initial  $b'$  abundance. Even if we let  $r_0$  range between  $1 - 10^{-10}$ , by taking  $h = 0.5$  we obtain  $\tau_{b'} \leq 6 \cdot 10^{15} - 10^{16} \text{ s}$ . On the other hand, according to eq. (14), this bound translates into an upper limit

$$V_H \leq \frac{(m_{b'}/150 \text{ GeV})^{5/4}}{h^{1/4}(1 + 0.036 \lg r_0)^{1/4}} \cdot 3 \cdot 10^{11} \text{ GeV} \leq 4 \cdot 10^{11} \text{ GeV} \quad (16)$$

where we have taken the conservative upper bound  $m_{b'} < 150 \text{ GeV}$  from the analysis [7]. For more realistic values  $m_{b'} \simeq 100 \text{ GeV}$  [7] we get  $V_H \leq 2.4 \cdot 10^{11} \text{ GeV}$ . According to eq. (12), this upper limit on the scale  $V_H = V_3$  together with the experimental limit  $m_{\nu'} \geq M_Z/2$  translates into a lower bound on the  $\tau$ -neutrino mass:

$$m_{\nu_\tau} \simeq h_N^{-1} \frac{m_{\nu'}^2}{V_H} \gtrsim (1 - 10) \text{ eV} \quad (17)$$

where in the numerical estimate we have taken into account the  $O(1)$  uncertainties in the Yukawa coupling  $h_N$  (for perturbativity we have to assume  $h_N \leq 3$ ). On the other hand, the most conservative cosmological upper bound on the light stable neutrino masses  $m_{\nu_\tau} \leq 92 \Omega h^2 \text{ eV}$  [14] provides a lower bound on  $V_H$ :

$$V_H \geq \frac{h_N^{-1}}{4\Omega h^2} \left( \frac{m_Z^2}{92 \text{ eV}} \right) \gtrsim (0.3 - 1) \cdot 10^{11} \text{ GeV} \quad (18)$$

More stringent limits on  $r_0$  and  $\tau_{b'}$  can be derived by considering that the late decay of the  $b'$  can cause a significant contribution to observed cosmic ray fluxes, in particular to the isotropic diffuse gamma-ray background [17]. Indeed, at the moment of decay, the  $b'$  quarks are bounded within colorless hadrons like  $b'ud$  or  $\bar{b}'u$  [15]. Then the decay  $b' \rightarrow b\bar{\nu}_\tau\nu'$  will produce an hadronic jet with the  $b$  quark being the leading particle and an excitation energy  $E_0 \simeq \frac{1}{3}m_{b'}$ . The fragmentation of this jet produces  $\pi^0$ ,  $\eta$  etc., with the subsequent radiative decay resulting in a specific photon spectrum, and the number of photons produced is directly proportional to  $r_0$ . In order to estimate the flux at the

present era, the redshift in the photon energies has to be taken into account as well. As long as the decay happens at the matter dominated epoch, and the small amount of relativistic decay products does not affect sensibly the Universe expansion rate, we have  $1 + z = (t_0/\tau_{b'})^{2/3} \sim 10 - 20$  for the values of  $\tau_{b'}$  estimated above. To compute the value of the isotropic cosmological gamma-flux we need to know what fraction of the jet energy  $E_0$  is taken by the photons and what is the energy spectrum. The photon spectra produced in jet hadronization for different leading particles were computed, using a Monte Carlo simulation program, in ref. [18]. These spectra exhibit a remarkable scaling property in terms of the variable  $x = E_\gamma/E_0$ , and in the case the leading particle is a  $b$  quark, the photons carry away about 25 percent of the initial jet energy. Using the results of ref. [18] we have estimated the present value of the gamma-flux  $d\Phi_\gamma/dE_\gamma$  as a function of  $r_0$ , and we have compared it with the existing observational limits (see [19] and references therein). For example, for  $E_\gamma = 100$  MeV the experimental bound is  $d\Phi_\gamma/dE_\gamma \lesssim 10^{-7} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \text{ MeV}^{-1}$  and the cosmic gamma-flux produced from  $b'$  decays at  $z = 10 - 20$  saturates this bound for  $r_0 \sim 10^{-7}$ .<sup>6</sup> Thus, for the preferred values  $r_0 \sim 10^{-7} - 10^{-8}$  [15] our model is consistent with the observational data on the isotropic gamma-background, and can even provide an appealing explanation of its origin.

The previous analysis implies that  $r_0$  should be rather small. In particular this excludes any sizeable baryon asymmetry between  $b'$  and  $\bar{b}'$ . Then the possible baryogenesis mechanisms applicable to our model are severely constrained. Even if a baryon asymmetry in the fourth family is hardly expected, since there is no mixing with the other three families, and hence no source of CP violation, the high rate of sphaleron processes before the electroweak phase transition [20] would immediately redistribute any baryon asymmetry present in the first three families to the fourth one. Therefore, no mechanism is acceptable which generates the baryon asymmetry before the sphaleron effects are switched off.<sup>7</sup> In the context of our model, the most appealing scenario is to assume that baryon asymmetry is generated at the electroweak (first order) phase transition, as a result of CP violation in the dynamics of quarks or leptons interacting with the walls of the expanding bubbles of the broken phase [23]. Outside of the bubbles the electroweak symmetry is unbroken, quarks are massless and the rate of the fermion number violation due to sphaleron transitions greatly exceeds the Universe expansion rate. Inside the bubbles the quarks are massive due to non-zero VEVs of the Higgs fields, the sphaleron processes are strongly suppressed and fermion number is effectively conserved. Baryon asymmetry inside the bubbles could be produced (and maintained) due to CP violating effects, as a difference between the quark and anti-quark fluxes penetrating the walls from the unbro-

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<sup>6</sup> Substantially larger  $r_0$  would require much larger redshift, and hence much smaller  $\tau_{b'}$ . However, the lower bound (18) on the breaking scale  $V_H$  excludes much smaller lifetimes.

<sup>7</sup>In principle, in our model the baryogenesis with non-zero B-L could occur due to CP violation effects in out-of-equilibrium decays  $N^c \rightarrow l + \Phi$  of the heavy right-handed neutrino [21] (for the viability of this mechanism in the SUSY case see ref. [22]), or in the decays of  $SU(4)_H$  gauge or scalar bosons. Then sphaleron effects would immediately transfer the produced net lepton number into a baryon asymmetry also in the fourth family sector. However, our model naturally avoids the possibility of such a leptobaryogenesis. As it was shown in ref. [22], the large scale density fluctuations hinted by the COBE measurements require rather low inflationary reheat temperature ( $T_R \sim 10^8$  GeV) and correspondingly low inflaton mass ( $m_\eta \sim 10^{11}$  GeV). Then for  $V_H \sim 10^{11}$  GeV the masses of *all* right-handed neutrinos and horizontal bosons are  $\gtrsim m_\eta$ , and therefore they cannot be produced after inflation.

ken phase to the broken one. This will affect only the first three family fermions. Since the fourth family is unmixed and has no CP violation, no baryon excess is expected in this sector. Although the viability of such a baryogenesis in the SM is still disputed in the literature, in the context of SSM it could be more effective and sufficient for providing the observed baryon asymmetry. Clearly this topic deserves additional special considerations.

Few remarks about the neutrino masses are now in order. For the  $\tau$ -neutrino mass our model predicts the lower bound eq. (17) of a few eVs, and according to (13)  $\nu_\mu$  and  $\nu_e$  are expected to have much smaller masses. A  $\nu_\tau$  with mass in the range 1 – 10 eV will give a sizeable contribution to the cosmological energy density as a hot dark matter (HDM) component. We remind here that the COBE measurements of the cosmic microwave background anisotropy, together with other data on the density distribution of the Universe at all distance scales (galaxy-galaxy angular correlations, correlations of galactic clusters, etc.), can all be fit by assuming some HDM admixture to the dominant CDM component [24]. The best fits hint to a neutrino mass  $m_{\nu_\tau} \sim 5 - 7$  eV [25] which does appear naturally in our model. As for the CDM itself, in our R parity conserving SUSY model it can be naturally provided by the lightest supersymmetric particle (LSP), presumably a neutralino.

As we commented earlier, the neutrino mass hierarchy should be qualitatively the same as that for the charged quarks and leptons. However, the spread in the Yukawa coupling constants  $h_F$  does not allow to put severe limits on the other neutrino masses. For example, by taking  $m_{\nu_\mu}/m_{\nu_\tau} \sim m_c/m_t$ , as is suggested by the first estimate in eq. (13), one obtains  $m_{\nu_\mu} \sim (2 - 5) \cdot 10^{-3}$  eV. This range corresponds to the MSW solution of the solar neutrino problem [26] via  $\nu_e \rightarrow \nu_\mu$  oscillations. Alternatively, if we had to attempt an explanation of the deficit of the atmospheric  $\nu_\mu$  via  $\nu_\mu \rightarrow \nu_e$  oscillations (for a recent analysis, see [27]), we would need  $m_{\nu_\mu} \sim 0.1$  eV which is compatible with the second estimate in eq. (13). Obviously the MSW explanation to the solar neutrino deficit would not be viable in this latter case.

## 4. Conclusions

In this paper we have put forward the idea that natural conservation of R parity in SUSY models can be guaranteed in the presence of a suitable horizontal gauge symmetry, so that an accidental  $Z_2$  matter parity (equivalent to R parity) automatically follows from gauge invariance and the field content of the model. On theoretical and phenomenological grounds, we have identified  $SU(4)_H$  as the only viable horizontal gauge group for implementing this idea. It implies existence of four fermion families. We have suggested a realistic SUSY model based on the SM vertical gauge group  $SU(3) \times SU(2) \times U(1)$  with an  $SU(4)_H$  anomaly free horizontal gauge symmetry. It leads to a particular form of the fermion mass matrices rendering all the fourth family fermions naturally in the 100 GeV range and unmixed with the first three families. This is due to the global  $\tilde{U}(1)$  symmetry which remains unbroken and in fact represents the conserved fermion number of the fourth family.<sup>8</sup> As for the masses of the light three families, our model leads to

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<sup>8</sup> Let us remark that actually this is a rather general statement which applies not only to our particular SUSY model designed for automatic R parity: if the fourth family fermions will be indeed discovered in

”seesaw” suppression of their magnitude from the electroweak scale down to the observed values. This suppression is achieved dynamically, without the need of any tuning of the Yukawa couplings which have been assumed to be all  $O(1)$ . By means of cosmological and astrophysical arguments, we have constrained rather precisely the scale  $V_H$  at which the horizontal gauge symmetry is completely broken, obtaining a narrow window around  $10^{11}$  GeV. The upper bound on the scale  $V_H$  sets a lower limit on the  $\tau$  neutrino mass of about few eV. Hence our model naturally provides cosmological HDM in the form of  $\nu_\tau$  and, due to R parity conservation, also CDM in the form of stable LSP. Since in our scheme conservation of R-parity is ensured by the horizontal gauge symmetry independently of the particular choice for the vertical gauge group, it would be interesting to extend the present analysis to phenomenologically appealing GUT models, such as  $SU(5)$  or  $E_6$ , for which R-parity conservation is not automatic.

## Acknowledgements

It is a pleasure to thank Venya Berezhinsky, Sasha Dolgov and Misha Shaposhnikov for illuminating discussions and useful comments.

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future colliders, it is not difficult to convince oneself that the only natural way of their accomodation is to to prescribe them the global conserved ”fourth flavour” number which renders the fourth neutrino to be a Dirac particle with  $O(M_W)$  mass.

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